Effect of Indicator Kriging on Recoverable Ore Reserves of El-Gidida Iron Deposit

M.A. Gouda & M.R. Moharam
Department of Mining and Petroleum Engineering, Al-Azhar University, Cairo, Egypt

ABSTRACT: Geological studies carried out on the El-Gidida iron deposit revealed that the ore body can be divided structurally into different zones. A three-dimensional geostatistical model was constructed for each zone with the aid of the powerful software "Vulcan". The objective of the present paper is to prove the efficiency of indicator kriging in estimating the recoverable ore reserves of the El-Gidida iron ore deposit above a certain cut-off grade. The recoverable ore reserves were estimated using the ordinary kriged model and the cumulative reserves were estimated without considering weighting factors. A comparison between cumulative ore reserves resulting from the indicator kriged model and ordinary kriged model was performed. The indicator approach helps to keep the tonnage above the cut-off grade approximately constant without distinct changes until the cut-off grade value is close to the average value of iron content. The tonnage above the cut-off grade, with the ordinary kriged model, started to decrease at lower iron content values.

1 INTRODUCTION

When constructing variograms, samples must belong to one zone in order to be able represent the spatial variability of that zone. These zones can be defined by studying the geology of the ore body, including its structural, mineralogical, and chemical properties. A database is then created for a specific zone which has the same geological features and similar assay values. Hence, vertical and horizontal variograms are constructed for each zone separately (Gouda et al., 1995).

The ore body of each zone was divided into three-dimensional small blocks of sizes 50 x 50 x 3 m where 50 x 50 m is the plan view and 3 m is the vertical thickness of each block. Ordinary kriging was used to calculate the estimated iron content of the small blocks to produce the three-dimensional kriged model for each zone.

When using the ordinary technique, the cumulative process was carried out without considering weighting factors. Depending on the grade/tonnage distribution of the iron content, the sum of the tonnage above each cut-off represents the cumulative tonnage.

In this case, the average grade was taken as the arithmetic mean of the grades above the cut-off. Indicator kriging was used to estimate the proportion of the ore body above a certain cut-off grade by considering weighting factors for the data above the cut-off grade. In this technique, indicator variograms were constructed, based on the available data which are above a specific cut-off grade, to be used with the indicator kriged model (Lemmer, 1984). Hence, the distribution, average grade and amount of the recovered tonnage at any cut-off grade can be obtained. In order to show the efficiency of indicator kriging, its cut-off grade-cumulative tonnage curves were compared with that produced from the ordinary kriged model.

2 GEOLOGY AND ZONES OF ORE BODY

The El-Gidida orebody is an oval-shaped depression and its main structural elements are a major anticline striking NE-SW and plunging to the NE, and normal faults trending NE-SW, N-S and NW-SE, as shown in Figure 1 (El-Aref & Lotfy, 1989). The ore body occurs more or less as a horizontal bed with a difference in elevation from roof to foot of up to 76 m. The ore body also includes lenses and lens-shaped intercalations, with thickness varying from 1 to 3 m and slightly mineralized barren rocks represented mainly by ferruginous clays and rarely by sands (El-Akkad & Issawi, 1963).

The structure geology of the ore body, where the presence of major faults reflects the possibility of dividing the ore body into seven zones, is shown in Figure 2. These zones are: C and M in the high
central area and NW, N, E, NE and SE in the wadi area. A computerized database was established for each zone based on grade information derived from 100 x 100 m test pits/drill hole grid system.

3 ORDINARY KRIGED MODELING

The ore body of each zone was divided into small blocks. The size was 50 x 50 x 3 m for each block (50 x 50 m through the horizontal direction and 3 m through the vertical direction).

3.1 Variograms

Variogram parameters are required in order to carry out kriged modeling. In this task, for each zone, vertical and horizontal variograms were constructed to express horizontal and vertical variability. These variograms were created for five zones: C, NW, M, SE and E where insufficient samples found in zones N and NE.

Table 1 illustrates the differences in the fitted variogram spherical model parameters: the nugget effect ($C_0$) and sill ($Q$) both horizontally and vertically, and the range of influence ($a$) vertically to prove the presence of zonal anisotropy within the ore body. The results indicate that there is a difference in the mineralization characteristics of each zone; hence, each zone must be modeled separately.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Vertical</th>
<th>Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_0$</td>
<td>$C$</td>
</tr>
<tr>
<td>C</td>
<td>21.9</td>
<td>73.8</td>
</tr>
<tr>
<td>NW</td>
<td>14.5</td>
<td>100</td>
</tr>
<tr>
<td>M</td>
<td>37.1</td>
<td>135</td>
</tr>
<tr>
<td>SE</td>
<td>18.9</td>
<td>92.4</td>
</tr>
<tr>
<td>E</td>
<td>19.4</td>
<td>90.8</td>
</tr>
</tbody>
</table>

3.2 Kriged models

The ordinary kriging technique was used to calculate the estimated iron content of the small blocks within the different zones to produce the three-dimensional kriged model for each zone (an example of these models is given in Figure 3).

<table>
<thead>
<tr>
<th>Zone</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>53.5</td>
<td>64</td>
<td>11.9</td>
</tr>
<tr>
<td>NW</td>
<td>42.9</td>
<td>7.5</td>
<td>17.7</td>
</tr>
<tr>
<td>M</td>
<td>48.6</td>
<td>6.6</td>
<td>13.6</td>
</tr>
<tr>
<td>SE</td>
<td>51.6</td>
<td>3.6</td>
<td>6.9</td>
</tr>
<tr>
<td>E</td>
<td>49.1</td>
<td>4.87</td>
<td>9.9</td>
</tr>
</tbody>
</table>
These models should also be used with the indicator kriging technique. The kriged models illustrate the distribution of iron content within the different zones, characterizing the poor and rich zones, as shown in Table 2.

The concept of dividing the ore body into different zones could be supported by this analysis where the statistical parameters clearly vary from one zone to another.

4 INDICATOR KRIGING

Indicator kriging allows the construction of a local or global histogram which can be considered unbiased estimators of precisely defined probabilistic distributions; hence, full account is given to the underlying continuity. Such estimates of local distributions can be used in mine planning for the estimation of local recoverable reserves (Journel, 1984).
4.1 Indicator variograms

Ordinary kriging requires a model of the variogram of the variable being estimated, which is iron content in our case. Indicator kriging requires an indicator variogram, which is different from the variogram used with ordinary kriging.

The indicator variogram depends on the determining of the proportion of iron content values above each cut-off grade \( z \). Each one of the iron content values can be transformed into an indicator by considering different weights for the proportions below and above the cut-off grade (Ishak & Srivastava, 1989). The indicator variogram can be defined as (Journel, 1982):

\[
\gamma(h,z) = 0.5E\{I(x+h,z) - I(x,z)\}^2
\]

Indicator kriging can be performed at several cut-offs using a separate variogram model for each cut-off. In the present study, an approximation is introduced to indicator kriging by using the same variogram model for estimation at all cut-offs. The variogram model chosen for all the cut-offs is developed from the indicator data at a cut-off close to the median as defined in statistics. It is the middle value of a set of numbers arranged in order of magnitude. Indicator variograms can be constructed based on the introduced approximation. Since the variogram used in this approximation to indicator kriging is based on the median indicator, it is usually referred to as "median indicator kriging".

Table 3 presents the fitted indicator variogram spherical model parameters to be used in indicator kriging modeling. These indicator variogram parameters seem to have different values from those of the variogram used with ordinary kriging due to the characteristics of the indicator approach. The deduced ranges of influence are less than those used with ordinary kriging and vary from one zone to another due to the cut-off grade taken into account. A nugget effect was also found in indicator variogram models within different zones, confirming that the ore body of the El-Gidida area is characterized by a wide variety of mineralogical composition and distribution of main ore-bearing and gangue minerals.

<table>
<thead>
<tr>
<th>Table 3. Indicator variogram parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>NW</td>
</tr>
<tr>
<td>M</td>
</tr>
<tr>
<td>SE</td>
</tr>
<tr>
<td>E</td>
</tr>
</tbody>
</table>

4.2 Indicator kriged models

Indicator kriging (IK in short) was introduced to infer a model for the conditional cumulative density function (ccdf), e.g., the recovered tonnage and quantity of metal above the cut-off (Perez, 1988).

DC introduces indicator step functions in the grade \( z(x) \) at \( x \) with the mean value over an area \( A \) within a deposit \( D \):

\[
\phi(A,z) = \frac{1}{A} \int_A I(x,z)dx
\]

The proportion of the tonnage recovered and the corresponding recovered quantity of metal after applying cut-off \( z \) is given by:

\[
l(A,z) = 1 - \phi(A,z)
\]

The integral (2) can be accomplished by numerical integration since all indicator values are known in the deposit \( A \):

\[
\phi(A,z) = \frac{1}{|A|} \sum_{i=1}^{n} I(x_i,z)
\]

This equation can be approximated by the weighted linear combination:

\[
\phi(A,z) = \frac{1}{|A|} \sum_{i=1}^{n} w(x_i,z) I(x_i,z)
\]

If \( n \) samples are available over \( A \), the weights \( w(x_i,z) \) are defined according to some criterion. For example, the polygonal method can be used to determine these weights. In this case, the weights are proportional to the polygonal area of each sample location \( x_i \).

Indicator kriging should be accompanied by ordinary kriging. In this case, two groups of variogram parameters were used. In the first, the deduced variogram parameters listed in Table 1 were considered with ordinary kriging, and in the second group, the indicator variogram parameters recorded in Table 3 were taken into account.

The indicator kriging technique was developed to calculate the cumulative tonnage above each cut-off. The cumulating process is dependent on considering the weights at each cut-off. The three-dimensional indicator kriged models (an example is given in Figure 4) illustrate the distribution of the iron content within the ore body after taking a specific cut-off grade.

The richest parts of the ore body of each zone can also be identified through indicator kriged models at higher cut-offs. Zones C and M contain the largest amounts of rich ore in comparison with the other
was carried out depending on the grade/tonnage for the blending process.

These values are approximately as the average value zone M, 55% for zone SE and 50% for zone E.

the cut-off grade, when indicator kriging was used, arithmetic mean of the grades above the cut-off. The sum of tonnage above each cut-off represents the cumulative distribution of the iron content. The sum of tonnage from indicator kriging (Figure 5).

In the ordinary technique, the cumulative process weighting factors into account) and that derived from indicator kriging (Figure 5).

5 CUT-OFF GRADE TONNAGE CURVES
In order to illustrate the effect of considering weights when indicator kriging is used and its efficiency in estimating the recoverable tonnage, a comparison was made between the recovered tonnage, at different cut-offs for each zone, derived from the ordinary kriged models (i.e., without taking weighting factors into account) and that derived from indicator kriging (Figure 5).

5.1 Cumulative tonnage
In the ordinary technique, the cumulative process was carried out depending on the grade/tonnage distribution of the iron content. The sum of tonnage above each cut-off represents the cumulative tonnage and the average grade was taken as the arithmetic mean of the grades above the cut-off.

It is obvious that the cumulative tonnage above the cut-off grade, when indicator kriging was used, gave better results than the ordinary kriging technique. The cumulative tonnage above the cut-off does not clearly decrease until it reaches about 55% cut-off for zone C, 45% for zone NW, 50% for zone M, 55% for zone SE and 50% for zone E. These values are approximately as the average value of the iron content within the different zones. However, the resulting cumulative tonnage above the cut-off starts to decrease clearly at about 40%, 30%, 35%, 45% and 40% cut-off grades with ordinary kriging. The recoverable ore reserve of El-Gidida ore body could change according to the technique used to estimate it. Dividing each zone into small three-dimensional blocks and considering weighting factors with the indicator approach made the high iron content values play an important role in keeping the tonnage above the cut-off approximately constant until the grade value was close to the average.

This reflects the importance of considering weighting factors when the cumulative process is performed as in indicator kriging to overcome the wide variability of iron content within the El-Gidida ore body. Consequently, the ability to control the mine planning and production processes could be improved. In addition, indicator kriging could enable the use of rich and poor parts of the ore body.

5.2 Relationship between average grade and cut-offs
It is important to show how the average iron content of the recoverable tonnage at each cut-off is affected by the application of indicator kriging. As shown in Figure 6, the average iron content within the different zones seems to be higher with indicator kriging than with ordinary kriging. This is another way to prove the efficiency of the indicator technique in calculating the cumulative tonnage.

Calculating the average grade on the basis of taking weights for the different cut-offs into account revealed that the low iron content values have only a small effect on the recoverable ore estimate, i.e., the low iron content values do not represent any major problem when determining such an estimate.

6 CONCLUSIONS
This following conclusions can be made from this study:
1. The vertical and horizontal variogram parameters showed that the ore body has zonal anisotropy and it should be not be considered as one zone so as to avoid a lack of information on its mineralization characteristics.
2. The ordinary kriged modeling of the iron content showed the importance of using three-dimensional kriged modeling where the horizontal and vertical variability are taken into account at the same time.
3. The use of indicator kriging is important in calculating cumulative tonnages of the El-Gidida iron deposit according to a cut-off grade.
4. The cumulative tonnage within the different zones does not decrease clearly until the grade value is close to the average value due to the distribution of the three-dimensional values of iron content.
5. The low iron content values do not have a clear effect on the cumulative tonnage where high values tend to have high weighting factors, giving high tonnage above a high cut-off.
6. Indicator kriged models could be used for the blending process to provide the charge to the blast furnace at a constant rate of average iron content and to use poor and rich mineralized parts of the ore body.
7. This approach may solve problems due to the high variability and mineralization characteristics of the ore body.

REFERENCES
Application of Computers and Operations Research in the Mineral Industry (APCOM), Johnson, T. and Bames, R (editors), pp 793-806


Perez, VS, 1988 Indicator knging based on principal component anal>sts M Sc thesis, Department of Applied Earth Sciences, Stanford University


Figure 5. Relationship between cut-off grade and tonnage above it within different zones.
Figure 6  Relationship between cut-off grade and average grade within different zones